

HW Help for 5/1

Multiply the numerator *and* denominator by the conjugate of the expression containing the square root.

The *conjugate* of a two-term expression is just the same expression with subtraction switched to addition or vice versa.

For this problem, you multiply the top and the bottom by the conjugate of $\sqrt{x} - 2$, which is $\sqrt{x} + 2$.

$$\begin{aligned} & \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \\ &= \lim_{x \rightarrow 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x - 4)(\sqrt{x} + 2)} \end{aligned}$$

The product of conjugates is always the square of the first thing minus the square of the second thing.

$$\begin{aligned} &= \lim_{x \rightarrow 4} \frac{\sqrt{x}^2 - 2^2}{(x - 4)(\sqrt{x} + 2)} \\ &= \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)} \end{aligned}$$

Cancel the $(x - 4)$ from the numerator and denominator.

$$= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2}$$

Now substitution works.

$$\begin{aligned} &= \frac{1}{\sqrt{4} + 2} \\ &= \frac{1}{4} \end{aligned}$$

$$\text{So, } \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \frac{1}{4}.$$