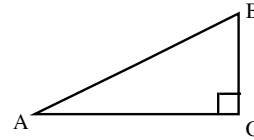


RIGHT TRIANGLE TRIGONOMETRY

#9

The three basic trigonometric ratios for right triangles are the sine (pronounced "sign"), cosine, and tangent. Each one is used in separate situations, and the easiest way to remember which to use when is the mnemonic SOH-CAH-TOA. With reference to one of the acute angles in a right triangle, Sine uses the Opposite and the Hypotenuse - SOH. The Cosine uses the Adjacent side and the Hypotenuse - CAH, and the Tangent uses the Opposite side and the Adjacent side -TOA. In each case, the position of the angle determines which leg (side) is opposite or adjacent. Remember that opposite means "across from" and adjacent means "next to."



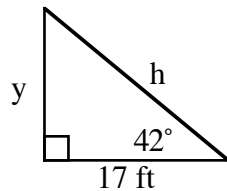
$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC}$$

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB}$$

Example 1

Use trigonometric ratios to find the lengths of each of the missing sides of the triangle below.



The length of the adjacent side with respect to the 42° angle is 17 ft. To find the length y , use the tangent because y is the opposite side and we know the adjacent side.

$$\begin{aligned}\tan 42^\circ &= \frac{y}{17} \\ 17 \tan 42^\circ &= y \\ 15.307 \text{ ft} &\approx y\end{aligned}$$

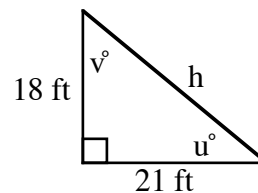
The length of y is approximately 15.31 feet. To find the length h , use the cosine ratio (adjacent and hypotenuse).

$$\begin{aligned}\cos 42^\circ &= \frac{17}{h} \\ h \cos 42^\circ &= 17 \\ h &= \frac{17}{\cos 42^\circ} \approx 22.876 \text{ ft}\end{aligned}$$

The hypotenuse is approximately 22.9 feet long.

Example 2

Use trigonometric ratios to find the size of each angle and the missing length in the triangle below.



To find $m\angle u$, use the tangent ratio because you know the opposite (18 ft) and the adjacent (21 ft) sides.

$$\begin{aligned}\tan u^\circ &= \frac{18}{21} \\ m\angle u^\circ &= \tan^{-1} \frac{18}{21} \approx 40.601^\circ\end{aligned}$$

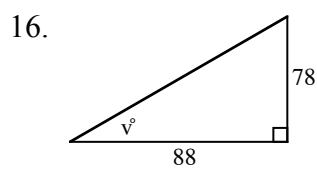
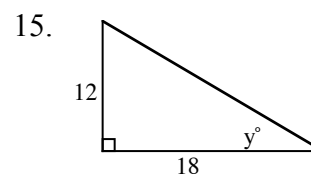
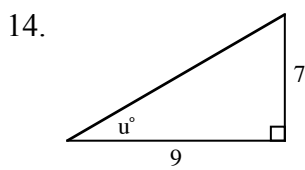
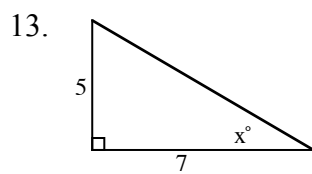
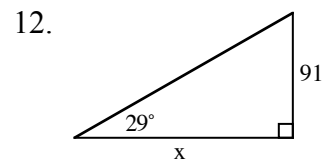
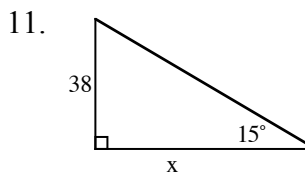
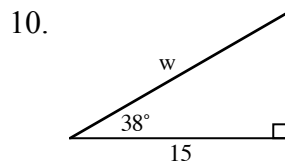
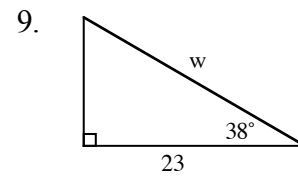
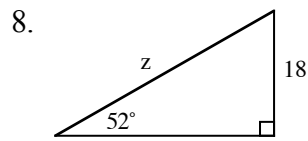
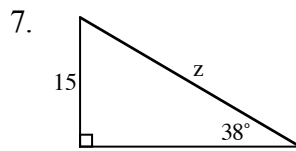
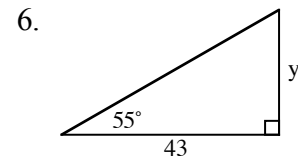
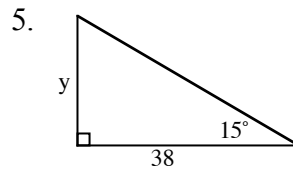
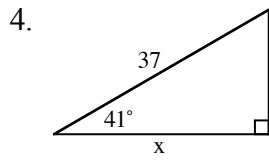
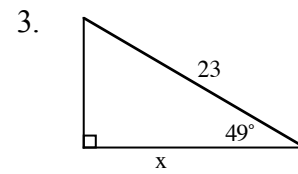
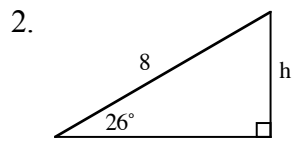
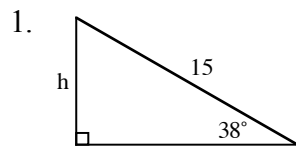
The measure of angle u is approximately 40.6° . By subtraction we know that $m\angle v \approx 49.4^\circ$.

Use the sine ratio for $m\angle u$ and the opposite side and hypotenuse.

$$\begin{aligned}\sin 40.6^\circ &= \frac{18}{h} \\ h \sin 40.6^\circ &= 18 \\ h &= \frac{18}{\sin 40.6^\circ} \approx 27.659 \text{ ft}\end{aligned}$$

The hypotenuse is approximately 27.7 feet long.

Use trigonometric ratios to solve for the variable in each figure below.



Draw a diagram and use trigonometric ratios to solve each of the following problems.

17. Juanito is flying a kite at the park and realizes that all 500 feet of string are out. Margie measures the angle of the string with the ground with her clinometer and finds it to be 42.5° . How high is Juanito's kite above the ground?
18. Nell's kite has a 350 foot string. When it is completely out, Ian measures the angle to be 47.5° . How far would Ian need to walk to be directly under the kite?
19. Mayfield High School's flagpole is 15 feet high. Using a clinometer, Tamara measured an angle of 11.3° to the top of the pole. Tamara is 62 inches tall. How far from the flagpole is Tamara standing?
20. Tamara took another sighting of the top of the flagpole from a different position. This time the angle is 58.4° . If everything else is the same, how far from the flagpole is Tamara standing?

Answers

1. $h = 15 \sin 38^\circ \approx 9.235$ 2. $h = 8 \sin 26^\circ \approx 3.507$ 3. $x = 23 \cos 49^\circ \approx 15.089$

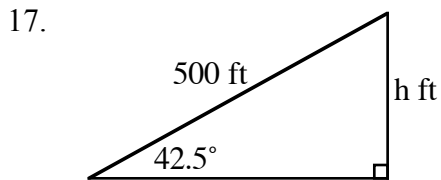
4. $x = 37 \cos 41^\circ \approx 27.924$ 5. $y = 38 \tan 15^\circ \approx 10.182$ 6. $y = 43 \tan 55^\circ \approx 61.4104$

7. $z = \frac{15}{\sin 38^\circ} \approx 24.364$ 8. $z = \frac{18}{\sin 52^\circ} \approx 22.8423$ 9. $w = \frac{23}{\cos 38^\circ} \approx 29.1874$

10. $w = \frac{15}{\cos 38^\circ} \approx 19.0353$ 11. $x = \frac{38}{\tan 15^\circ} \approx 141.818$ 12. $x = \frac{91}{\tan 29^\circ} \approx 164.168$

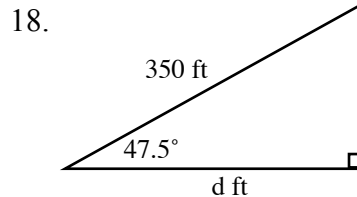
13. $x = \tan^{-1} \frac{5}{7} \approx 35.5377^\circ$ 14. $u = \tan^{-1} \frac{7}{9} \approx 37.875^\circ$ 15. $y = \tan^{-1} \frac{12}{18} \approx 33.690^\circ$

16. $y = \tan^{-1} \frac{78}{88} \approx 41.5526^\circ$



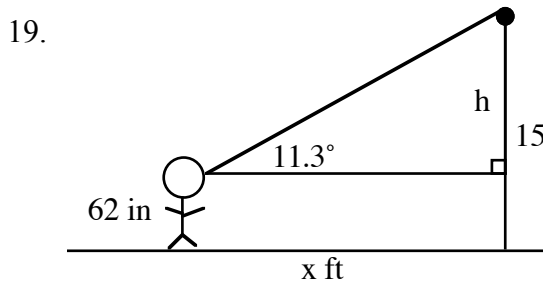
$$\sin 42.5 = \frac{h}{500}$$

$$h = 500 \sin 42.5^\circ \approx 337.795 \text{ ft}$$



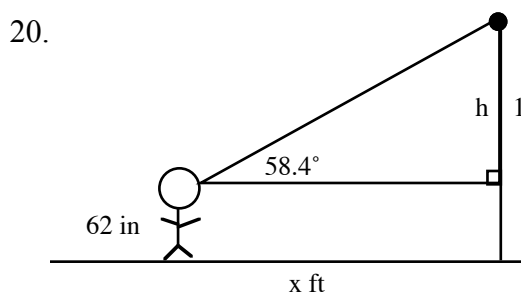
$$\cos 47.5^\circ = \frac{d}{350}$$

$$d = 350 \cos 47.5^\circ \approx 236.46 \text{ ft}$$



$$15 \text{ feet} = 180 \text{ inches}, 180'' - 62'' = 118'' = h$$

$$x \approx 590.5 \text{ inches or } 49.2 \text{ ft.}$$



$$h = 118'', \tan 58.4^\circ = \frac{118''}{x},$$

$$x \tan 58.4 = 118'', x = \frac{118''}{\tan 58.4^\circ}$$

$$x \approx 72.59 \text{ inches or } 6.05 \text{ ft.}$$